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THE AERODYNAMIC DESIGN OF SUPERSONIC PROPELLERS

FROM STRUCTURAL CONSIDERATIONS

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SUMMARY

The aerodynamic design of propellers from considerations of centrifugal force is presented. A solution is obtained for the spanwise distribution of cross-sectional area required to attain constant centrifugal stress over most of the blade. By applying a constant minimum value of thickness ratio from root to tip and allowing the distribution of required area to appear in the blade plan form, propellers with good efficiency are realized at high Mach numbers by the method of this paper.

Considerations of centrifugal stress show that the blade power-absorbing ability is seriously reduced with increasing flight Mach number. Increasing the design advance ratio is seen to cause a decrease in propeller efficiency and power absorption in the Mach number range from 1.2 to 2.4.

From a comparison of two propellers at a Mach number of 0.9 and an advance ratio of 2.0, both having the same allowable design stress and differing only in the manner in which the required area variation was applied, the blade having a constant minimum thickness ratio of 2 percent was found to be 7 percent more efficient than a rectangular propeller having the required area variation applied to thickness ratio with the minimum thickness ratio of 2 percent applied at the tip.

INTRODUCTION

Design procedure for subsonic propellers (for example, ref. 1) is based on the theory of Betz. Through the use of Betz's theory, induced losses are minimized. At transonic flight speeds, however, propeller blade sections operate in the supercritical region where large increases occur in profile-drag losses and where the induced losses are very low. Therefore, in order to obtain a supersonic propeller of highest efficiency, lift-drag ratios must be maximized so that the profile-drag losses are minimized. A reduction in profile-drag losses is chiefly accomplished with a supersonic propeller by decreasing the thickness ratio of the blade sections. The primary aerodynamic requirement for supersonic propellers, therefore,

is that they possess the thinnest possible sections. Sections of minimum
thickness can be achieved if each section along the blade works at the
allowable stress of the material.

A method is developed herein for obtaining the most efficient spanwise
distribution of cross-sectional area by considering only the effects of
centrifugal tensile force on a propeller blade. Once the cross-sectional-
area distribution is determined, the thinnest possible section can be
chosen to comply both with this distribution and with the requirement of
high lift-drag ratio. Although other stresses such as bending and torsion
(both steady and vibratory) may be equally important in some cases, they
do not alter the fact that centrifugal tensile force restricts the radial
cross-sectional-area distribution to a minimum. With the minimum cross-
sectional-area distribution established by centrifugal-force requirements,
modifications to the blade structure (if any) required by other stress
requirements can be safely made.

The aerodynamic characteristics of several propellers designed by
this method are studied at values of Mach number from 0.9 to 2.4.

The author would like to express appreciation to Messrs. Keith C. Harder
and Conrad Rennemann, Jr., of the Langley Aeronautical Laboratory for
their solution of the equations for constant centrifugal stress.

SYMBOLS

A   cross-sectional area of blade section; the subscript denotes
                station (for example, $A_{0.1}$ is cross-sectional area at
                $x = 0.1$), sq ft

b   blade width (chord), in.

c   speed of sound in air, ft/sec

F   centrifugal force, lb

D   propeller diameter, ft

h   blade section maximum thickness, in.

h/b   blade section thickness ratio

J   advance ratio, V/nD

M   Mach number of advance, V/c

n   propeller rotational speed, rps

P_c power disk-loading coefficient, $\frac{16}{J} Q_c$

Q   torque, ft-lb

Q_c torque disk-loading coefficient, $Q/\rho V^2 D^3$

R   propeller tip radius, ft

r   radius to a blade element, ft

T   thrust, lb

T_c thrust disk-loading coefficient, $T/\rho V^2 D^2$

V   velocity of advance, ft/sec

x   fraction of propeller tip radius, $\cdot r/R$

x_t radial station to which constant centrifugal stress extends

η   propeller efficiency, $T_c J/2\pi Q_c$

ρ   density of air, slugs/ft^3

μ   density of material, slugs/ft^3

σ   allowable design stress of material, lb/in.^{2}

ω   propeller rotational speed, radians/sec

BASIC CONCEPTS AND ASSUMPTIONS

        At transonic and supersonic forward speeds induced losses tend to
disappear. Comparisons of propellers on a profile-efficiency basis alone
are therefore warranted. For this reason, the assumption that the geometric
helix angle is equal to the aerodynamic helix angle is used throughout
this paper.

        In the subcritical speed range of propeller operation the choice of
optimum advance ratio from the standpoint of maximum section profile
efficiency is not critical inasmuch as it varies only slightly over a

wide range of advance ratios. At transonic and supersonic speeds, however, the regions of optimum advance ratios are more restricted. Operation at advance ratios of around 2.0 has been shown to be optimum (for example, see ref. 2).

Because high rotational speeds are necessary to maintain low advance ratios at high forward speeds, high levels of stress will be experienced in supersonic propellers. Steady stresses in a propeller blade are caused by centrifugal force, bending due to air loads, and torsion. Centrifugal stress assumes major importance for propeller operation at low advance ratios and high forward speeds. Through proper distribution of material along the blade, each section from root to tip can be made to operate at the design centrifugal stress. By so doing, every section of the pro- peller will be working at the same value of centrifugal stress, and such a propeller is termed herein a "constant stress" propeller. Based on these considerations, a solution for the cross-sectional-area distribution required for such a propeller is developed. In practice, of course, the centrifugal stress cannot be maintained constant along the entire blade because the stress must be zero at the tip.

In this paper a value of allowable stress equal to 50 percent of the ultimate strength has been assumed. This stress corresponds to material fatigue strength. In addition, this fatigue stress has been reduced by 20 percent to account for stresses other than centrifugal. Therefore, in this paper the term "allowable design" stress is equal to 40 percent of the ultimate strength of the material. Other stress limits could have been assumed equally as well, but these assumptions appear to be reason- able on the basis of past experience. The effect of the temperature rise due to aerodynamic heating on the strength of blade material has been neglected. Its influence would be felt as a lowering of the allowable design stress.

DEVELOPMENT AND SOLUTION OF CONSTANT-CENTRIFUGAL-STRESS EQUATION

For a propeller to operate at constant centrifugal stress, the centrifugal stress must be the same at every station. The value of this constant centrifugal stress shall be the allowable design stress of the material.

Consider a rotating beam and a section therein:
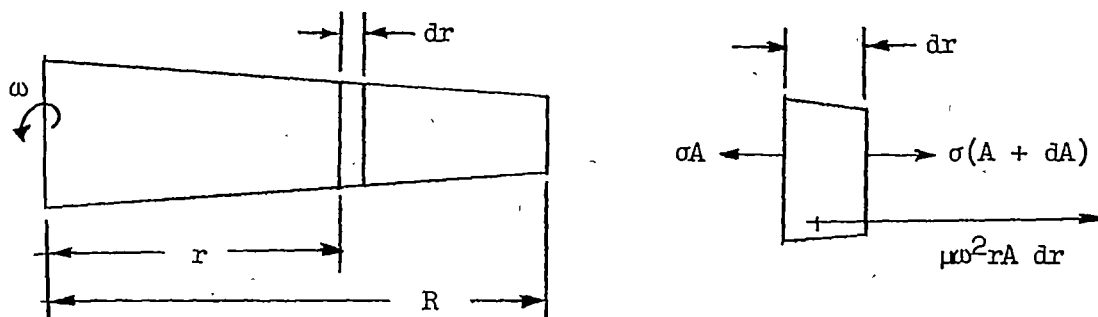
For equilibrium across the section the following equation must apply:

$$\sigma \, dA = -\mu\omega^2 rA \, dr$$

or

$$\sigma \frac{dA}{dr} = -\mu\omega^2 rA$$

Inasmuch as

$$r = Rx$$

the equation for equilibrium becomes

$$\sigma \frac{dA}{dx} = -\mu\omega^2 R^2 Ax \ .$$

The solution of this equation is

$$AC = e^{\frac{-\mu\omega^2 R^2 x^2}{2\sigma}}$$

where C is a constant of integration. In order to evaluate the constant C, let $A = A_0$ when $x = 0$. Then $C = \frac{1}{A_0}$ and A becomes

$$A = A_0 e^{\dfrac{-\mu\omega^2 R^2 x^2}{2\sigma}}$$

Now the force at any point x is

$$F = \sigma A_0 e^{\dfrac{-\mu\omega^2 R^2 x^2}{2\sigma}}$$

and at the tip, that is, at $x = 1$, the force is

$$F = \sigma A_0 e^{\dfrac{-\mu\omega^2 R^2}{2\sigma}}$$

This equation gives a finite, nonzero value for the force at the tip for every propeller of finite length. In other words, for this solution to apply, the propeller would have to be of infinite length. Hence, σ cannot be constant throughout a propeller of finite length.

In order to overcome this difficulty, the assumption is made that the stress is constant up to some point x_t and from that point is allowed to fall off to zero at the tip by holding the cross-sectional area constant from the point x_t to the tip. This assumption represents a practical configuration. As the point x_t is approached from the inboard direction, the force is

$$F = A\sigma$$

The force at x_t due to the mass beyond x_t is

$$F = \frac{\mu\omega^2 R^2}{2} A\left(1 - x_t^2\right)$$

To determine the point x_t such that both the force and area - and hence the stress - are continuous,

$$A\sigma = \frac{\mu\omega^2 R^2}{2} A\left(1 - x_t^2\right)$$

or

$$x_t^2 = 1 - \frac{2\sigma}{\mu\omega^2 R^2}$$

Hence, in the interval $0 \leqq x \leqq x_t$,

$$\sigma = \text{Constant} \qquad A = A_0 e^{\frac{-\mu\omega^2 R^2 x^2}{2\sigma}}$$

and in the interval $x_t \leqq x \leqq 1$,

$$\sigma = \frac{\mu\omega^2 R^2}{2} \left(1 - x^2\right)$$

and

$$A = \text{Constant} = A_0 e^{\frac{-\mu\omega^2 R^2 x_t^2}{2\sigma}}$$

where

$$x_t = \sqrt{1 - \frac{2\sigma}{\mu\omega^2 R^2}}$$

A more convenient form is

$$\frac{A_x}{A_0} = e^{-\frac{\mu}{2\sigma}\left(\frac{\pi V}{J}\right)^2 x^2}$$

up to the point where

$$x = x_t$$

where

$$x_t = \sqrt{1 - \frac{2\sigma}{\mu}\left(\frac{J}{\pi V}\right)^2}$$

from which point

$$\frac{A_x}{A_0} = e^{-\frac{\mu}{2\sigma}\left(\frac{\pi V}{J}\right)^2 x_t{}^2}$$

to the propeller tip.

This analysis offers no unique solution for blade-chord or thickness-ratio distribution but rather a solution for cross-sectional-area distribution. Combinations of thickness and chord may therefore be picked at will.

APPLICATION OF METHOD

In order to evaluate the influence of this method on the aerodynamic characteristics of propellers operating in the transonic and supersonic speed ranges, several propellers were designed and their characteristics are discussed.

Thickness ratio and pitch distribution.- As previously discussed, if propeller sections are to operate at the highest values of lift-drag ratio possible, they must possess minimum values of thickness ratio. Sections of 2-percent thickness ratio represent about the minimum with regard to fabrication at the present time, and in the following discussion solid blades with a thickness ratio of 2 percent from root to tip are assumed, unless otherwise noted. With a constant thickness ratio, the area distribution required by the condition of constant centrifugal stress appears as a variation in the blade plan form. This assumption by no means indicates that other conditions such as variation in thickness ratio or hollow blades could not be assumed. However, increases in thickness ratio lower the lift-drag ratios, and hollow blades pose manufacturing difficulties if the absolute thickness is small.

The pitch distribution incorporated into the propellers designed by the constant-stress method herein is such that each section along the blade works at a value of lift coefficient for maximum lift-drag ratio. These values of lift coefficient and lift-drag ratio were calculated from Ackeret's two-dimensional supersonic-flow theory modified to account for round-nose airfoils of finite thickness in a manner prescribed by W. F. Hilton in a British paper that is not generally available and to include skin friction. Although a pitch distribution incorporating lift coefficients for maximum lift-drag ratio may not be necessarily optimum, the use of such pitch distributions is satisfactory for aerodynamic comparisons of propellers designed by the method of this paper.

The propeller designs herein have equal diameters and root chords. (The term "root chord" herein pertains to the blade chord at the spinner juncture.) The choice of equal root chords results in propellers of identical hub and spinner configurations. The blades all produce the same centrifugal force - an important factor in the design of hubs for supersonic propellers.

Effect of allowable design stress on blade plan form.- Figure 1 presents a series of area-distribution curves for several values of allowable design stress. These curves are based on operation at a forward Mach number of 0.9 at an altitude of 40,000 feet and an advance ratio of 2.0. In order to obtain a blade plan form from any one of these curves, all that is required is to establish the spinner juncture or root chord and the thickness-ratio distribution and apply the area distribution shown to the rest of the blade. The point at which the area ratio becomes constant on each curve, labeled x_t, is the point to which constant centrifugal stress extends. Outboard of this point the stress decreases so that it is zero at the tip.

Figure 2 illustrates the effect of allowable design stress on tip-root chord ratios for steel propellers of constant thickness ratio at a Mach number of 0.9; for high allowable design stresses the tip-root chord ratios are seen to be reasonable. With materials of higher strength-weight ratio, such as titanium, these values of tip-root chord ratio would be further increased.

Effect of Mach number.- For the higher values of forward Mach number accompanied by higher values of rotational speed necessary to maintain low advance ratios, a corresponding decrease in tip-root chord ratios results. Area distributions, thrust and torque distributions, and corresponding efficiencies are shown in figures 3 to 9 for propellers at four Mach numbers and having an allowable design stress of 48,000 pounds per square inch and advance ratios of 2.0 and 3.0.

Figure 3 shows to what extent the plan form is changed with increasing Mach number for an advance ratio of 2.0. At the higher values of Mach number, most of the blade area has moved inboard. Although the scale of figure 3 is such as to indicate zero blade cross-sectional area when x is greater than about 0.75 for Mach numbers of 2.0 and 2.4, there is actually a small finite area. Figure 6 is a similar plot except at an advance ratio of 3.0. Here the same inboard shift of area is apparent, but to a lesser extent.

An examination of the elemental thrust and power coefficients, figures 4 and 5, respectively, for an advance ratio of 2.0 also shows the load to be concentrated inboard. This inboard shift is much less severe for an advance ratio of 3.0 (figs. 7 and 8). In fact, for the lowest Mach number analyzed, no shift is apparent. Propeller efficiency for advance ratios of 2.0 and 3.0 is plotted in figure 9. Good efficiencies

are realized for both advance ratios at the Mach numbers considered. At
a Mach number of 1.2, the efficiency is about the same in both cases;
whereas at higher Mach numbers operation at an advance ratio of 2.0 is
up to 3.5 percent more efficient. The calculations of propeller efficiency
reported herein made use of theoretical aerodynamic data. Induced losses
and shock losses due to blade interference were not considered. A com-
parison of power absorption between the two cases (fig. 10) shows that
the power absorption for an advance ratio of 2.0 is from 2 to 15 percent
greater than that for an advance ratio of 3.0. Thus, operation at an
advance ratio of 2.0 not only is more efficient but also enables greater
power absorption. Figure 10 also indicates a marked decrease in power
absorption with increasing Mach number for both values of advance ratio
considered.

Effect of varying the thickness ratio at an advance ratio of 2.0.-
A comparison between two extreme cases of constant-centrifugal-stress
propellers can be made - one with the required area variation applied
to blade chord, the other to blade thickness ratio. The blades were of
equal root chord at a radial station of 0.29 and of equal diameter, and
in both cases the material chosen was steel having an allowable design
strength of 48,000 pounds per square inch; the operating condition taken
was a forward Mach number of 0.9 and the advance ratio was 2.0. In the
first case the resulting plan form is shown in figure 11, and in the
latter case the plan form was rectangular, with a variation in thickness
ratio from 6 percent at the root to 2 percent at a radial station of 0.72,
thence 2 percent to the tip.

Propeller efficiency for the first propeller was calculated to be
85 percent, and for the second propeller the efficiency was calculated to
be 78 percent, a decrease in efficiency of 7 percent. However, the power
absorption of the rectangular propeller was 47 percent greater than that
of the tapered propeller.

CONCLUDING REMARKS

A solution for the variation in propeller-blade cross-sectional area
required to operate at a constant allowable value of centrifugal stress
over most of the blade has been obtained. Analyses in which a constant
value of thickness ratio of 2 percent is assumed indicate that propellers
with good efficiency can be obtained up to flight Mach numbers of at least
2.4 by using the method of this paper.

The design of propeller blades from considerations of centrifugal
stress shows that blade power-absorbing ability is seriously reduced with
increasing flight Mach number. Increasing the design advance ratio from
2.0 to 3.0 is shown to cause a decrease in propeller efficiency and power
absorption in the Mach number range from 1.2 to 2.4.

The importance of striving for low thickness ratios over the entire blade is illustrated by a comparison between two "constant centrifugal stress" propellers having the same design conditions and differing only in the manner in which the required area distribution is applied. This comparison shows that, at a Mach number of 0.9 and an advance ratio 2.0, a propeller with a constant thickness ratio of 2 percent having the required area variation appear in plan form is 7 percent more efficient than a rectangular propeller having the required area variation appear in thickness ratio with the minimum thickness ratio of 2 percent applied at the tip.

Langley Aeronautical Laboratory,
    National Advisory Committee for Aeronautics,
        Langley Field, Va., September 5, 1952.

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1. Hartman, Edwin P., and Feldman, Lewis: Aerodynamic Problems in the Design of Efficient Propellers. NACA ACR, Aug. 1942.

2. Smith, C. B., and Giarratana, S. A.: The Supersonic Propeller. Concluding Report. Rep. R-12004-03, United Aircraft Corp. Res. Dept., Apr. 23, 1948.
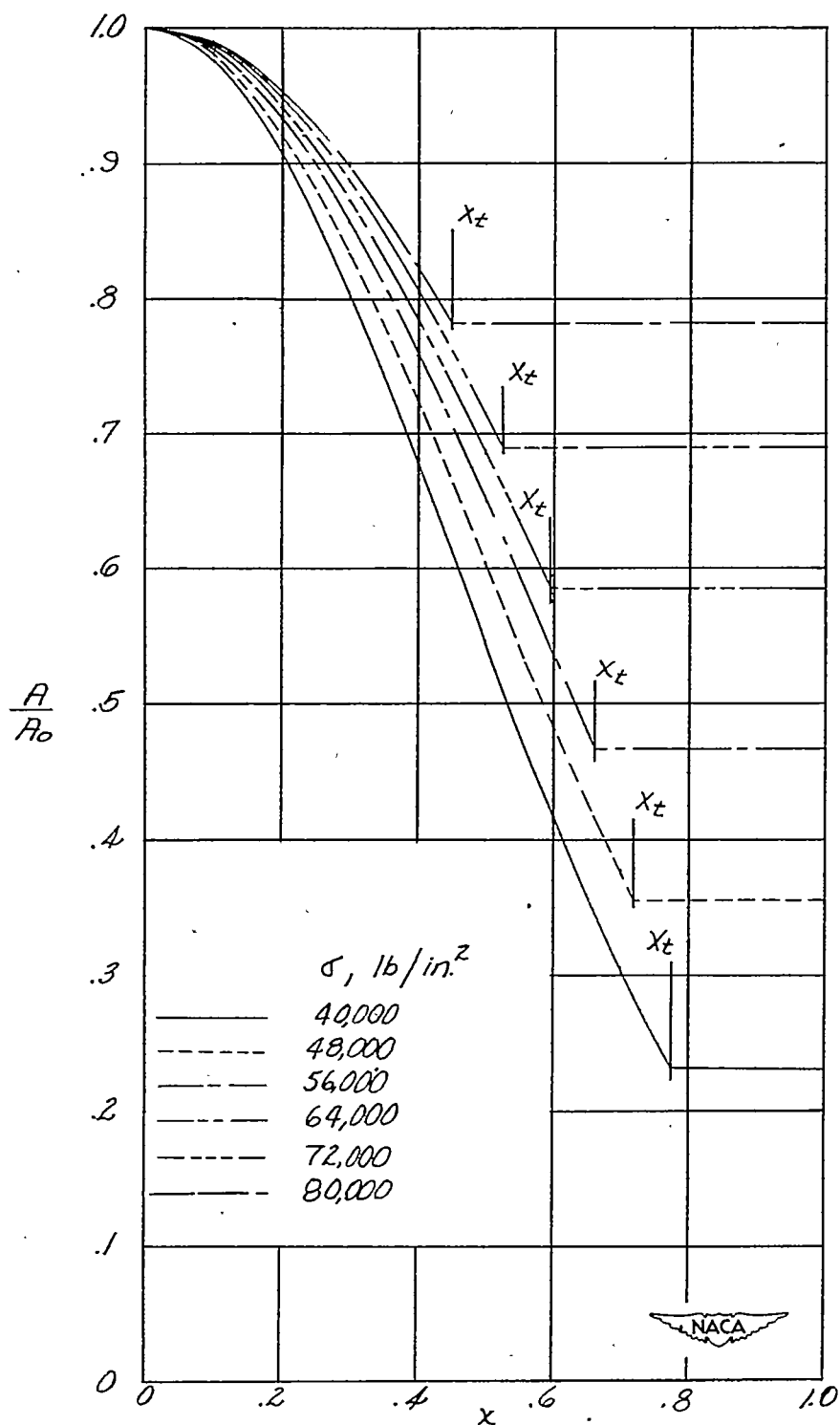
Figure 1.- Area distribution for steel propellers of constant centrifugal
stress. J = 2.0; altitude, 40,000 feet; M = 0.9.
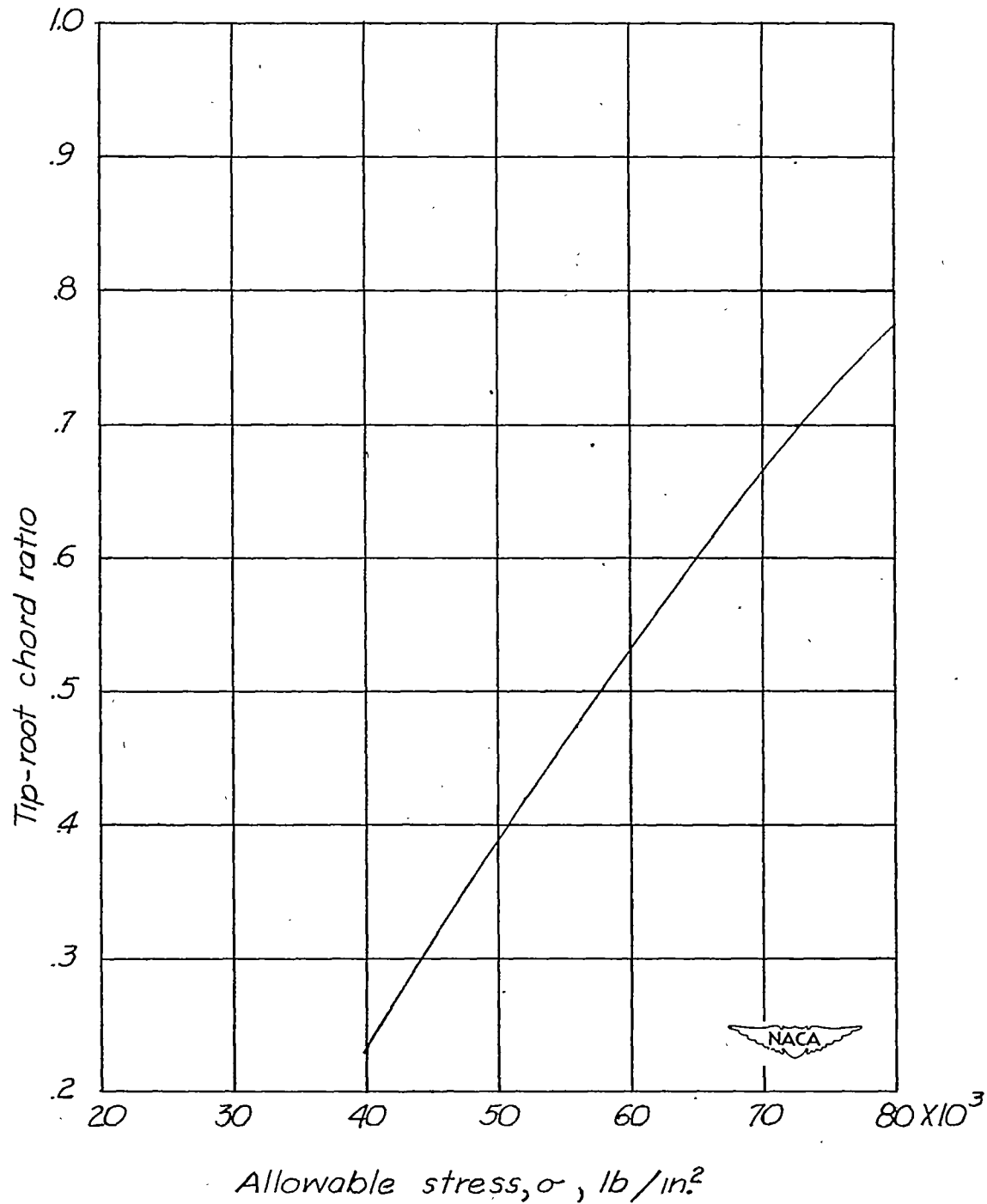
Figure 2.- Effect of allowable stress on tip-root chord ratios for steel propellers of constant centrifugal stress. $\frac{h}{b}$ = Constant; J = 2.0; M = 0.9; altitude, 40,000 feet.
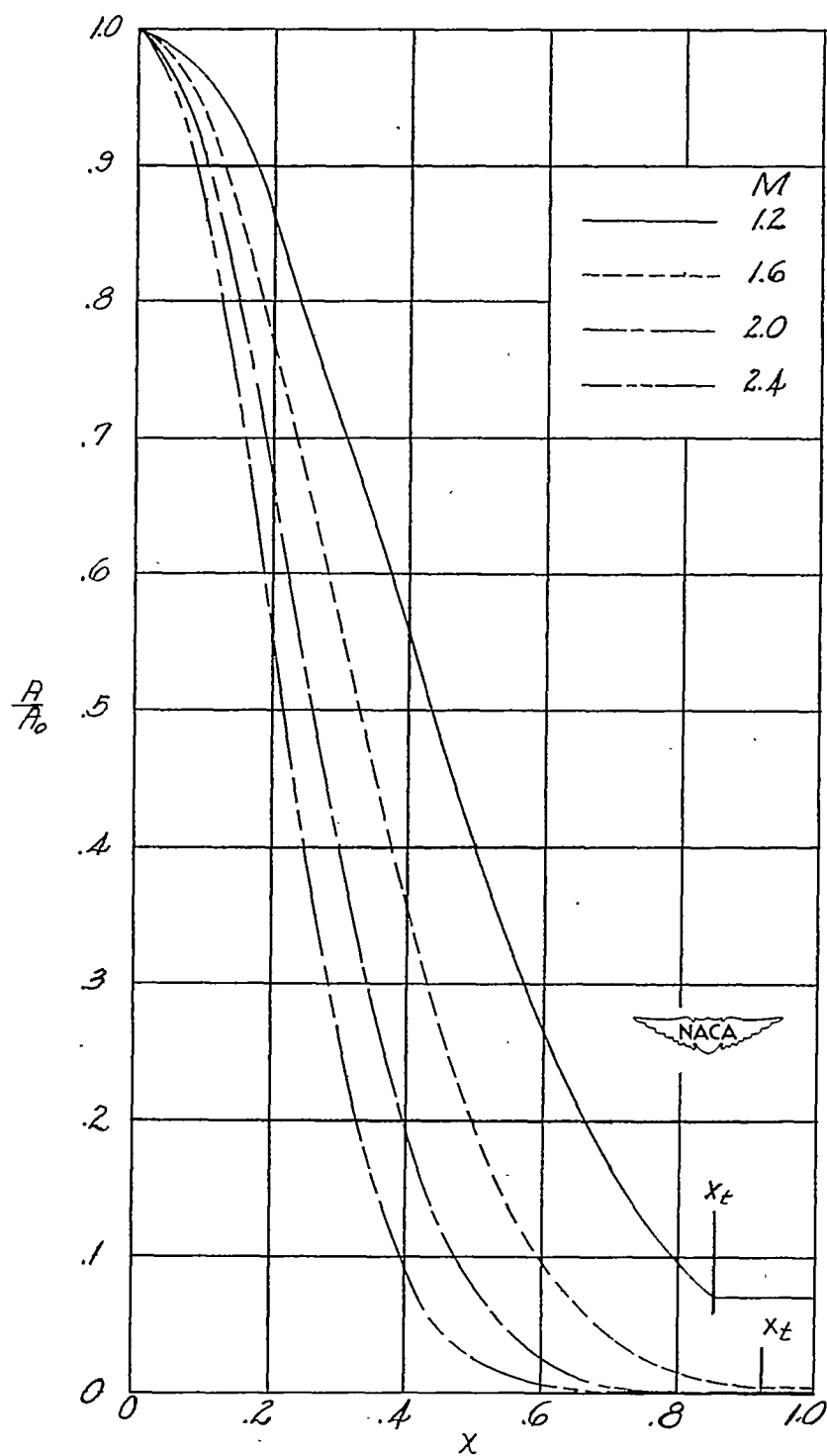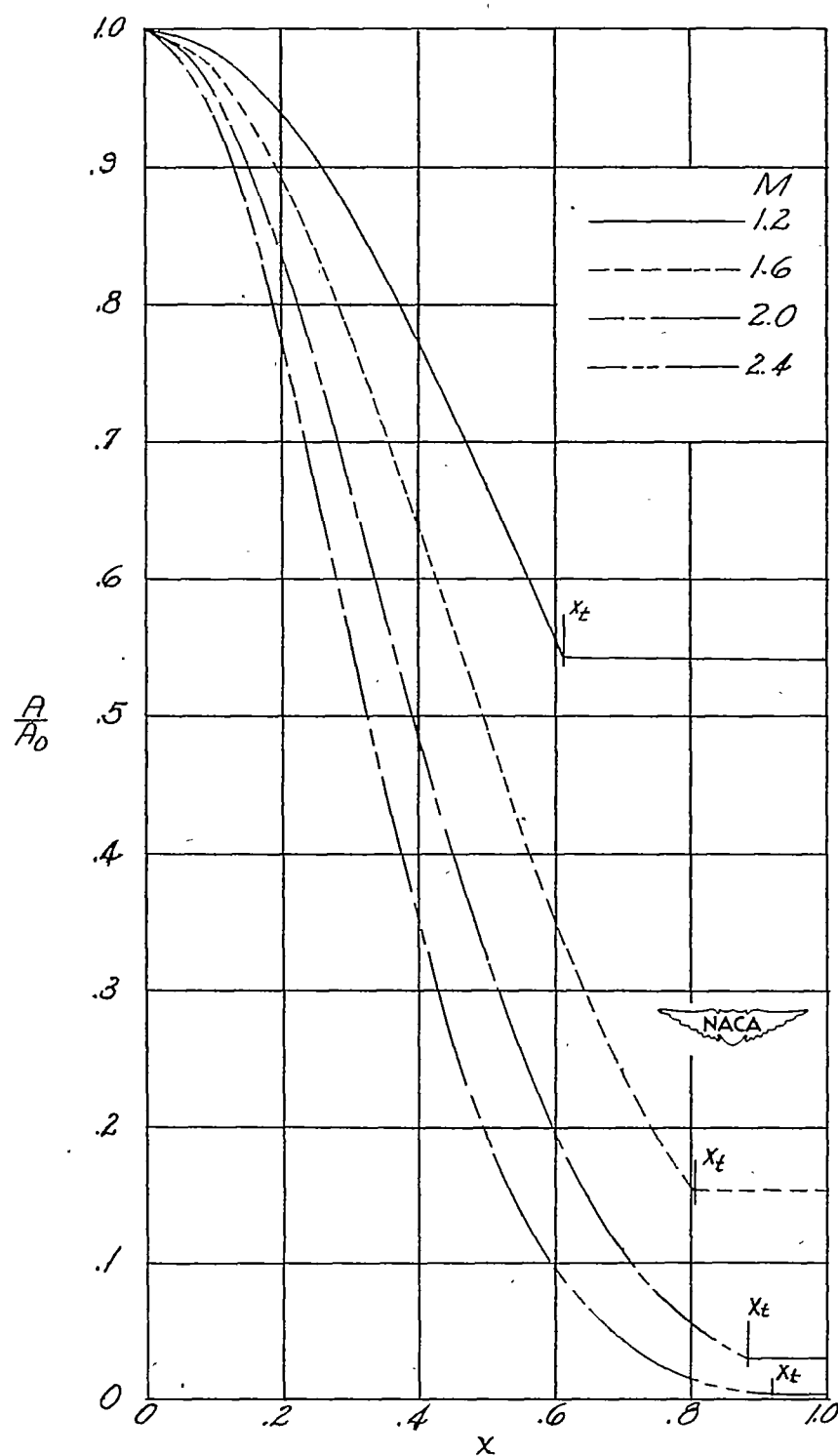
Figure 3.- Effect of Mach number on area distribution for steel
propellers of constant centrifugal stress at an advance ratio
of 2.0. σ = 48,000 pounds per square inch; altitude, 40,000 feet.
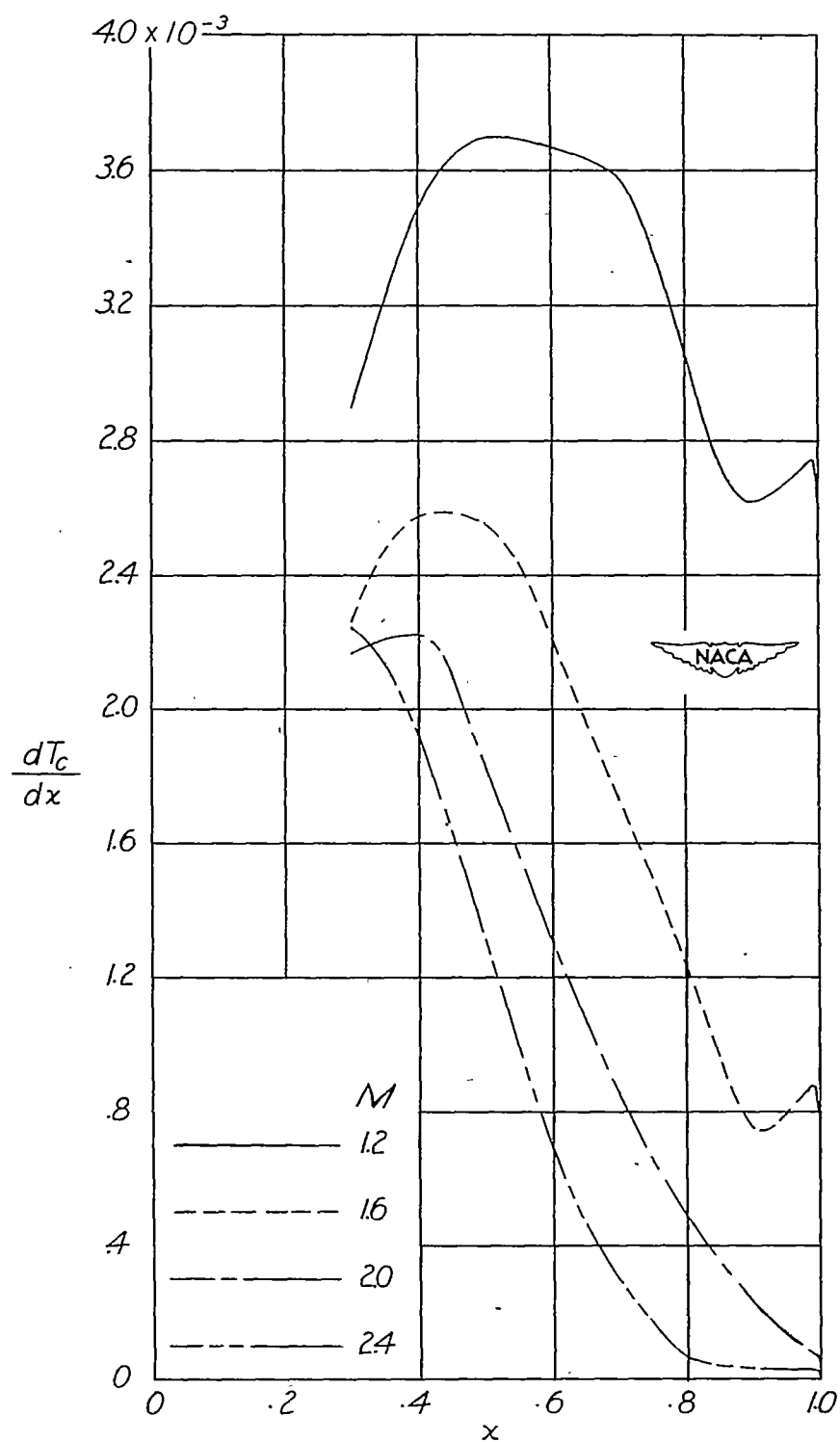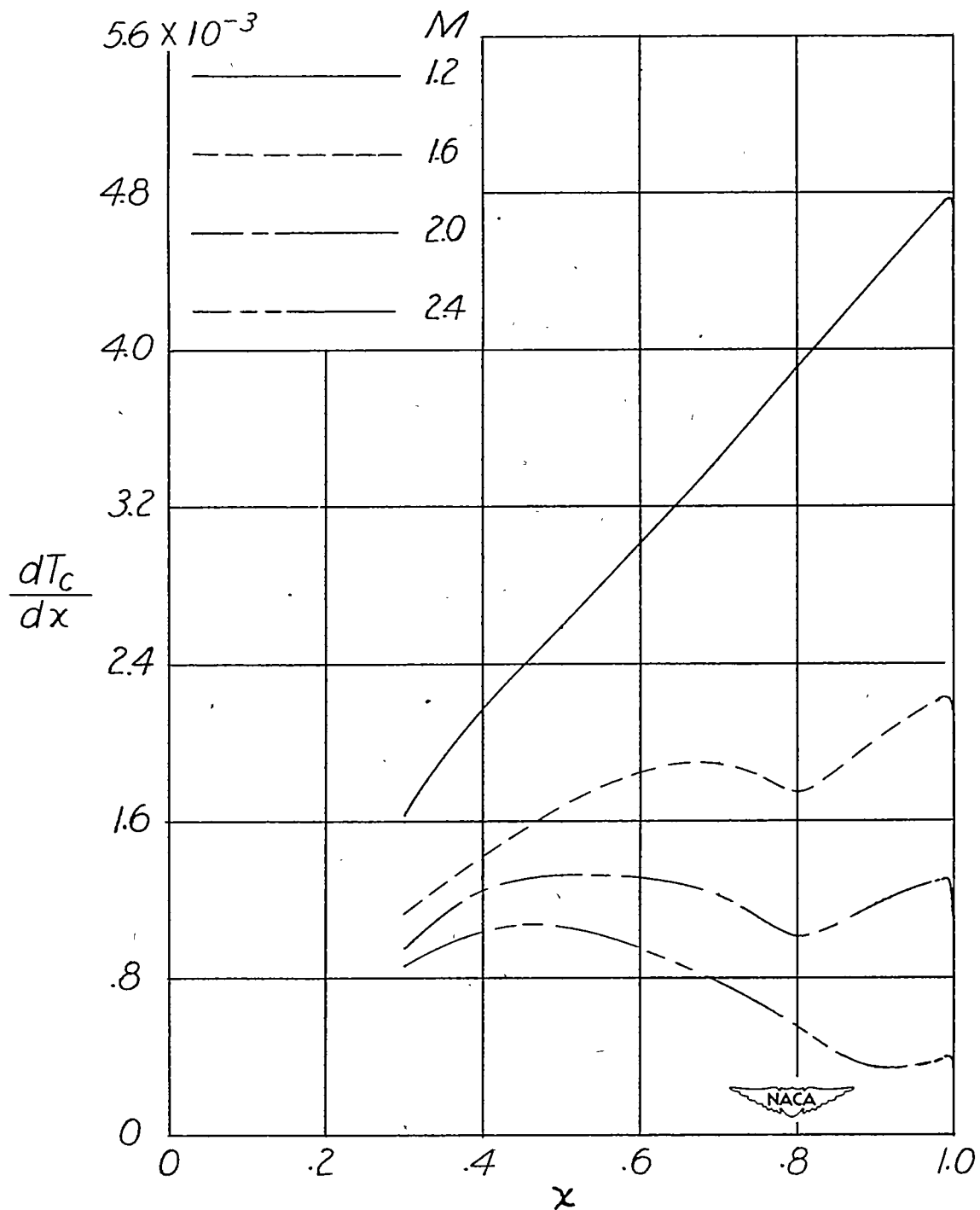
Figure 4.- Thrust-gradient curves for propellers of constant centrifugal stress at an advance ratio of 2.0. $\frac{h}{b} = 0.02$; altitude, 40,000 feet; $\sigma = 48,000$ pounds per square inch.
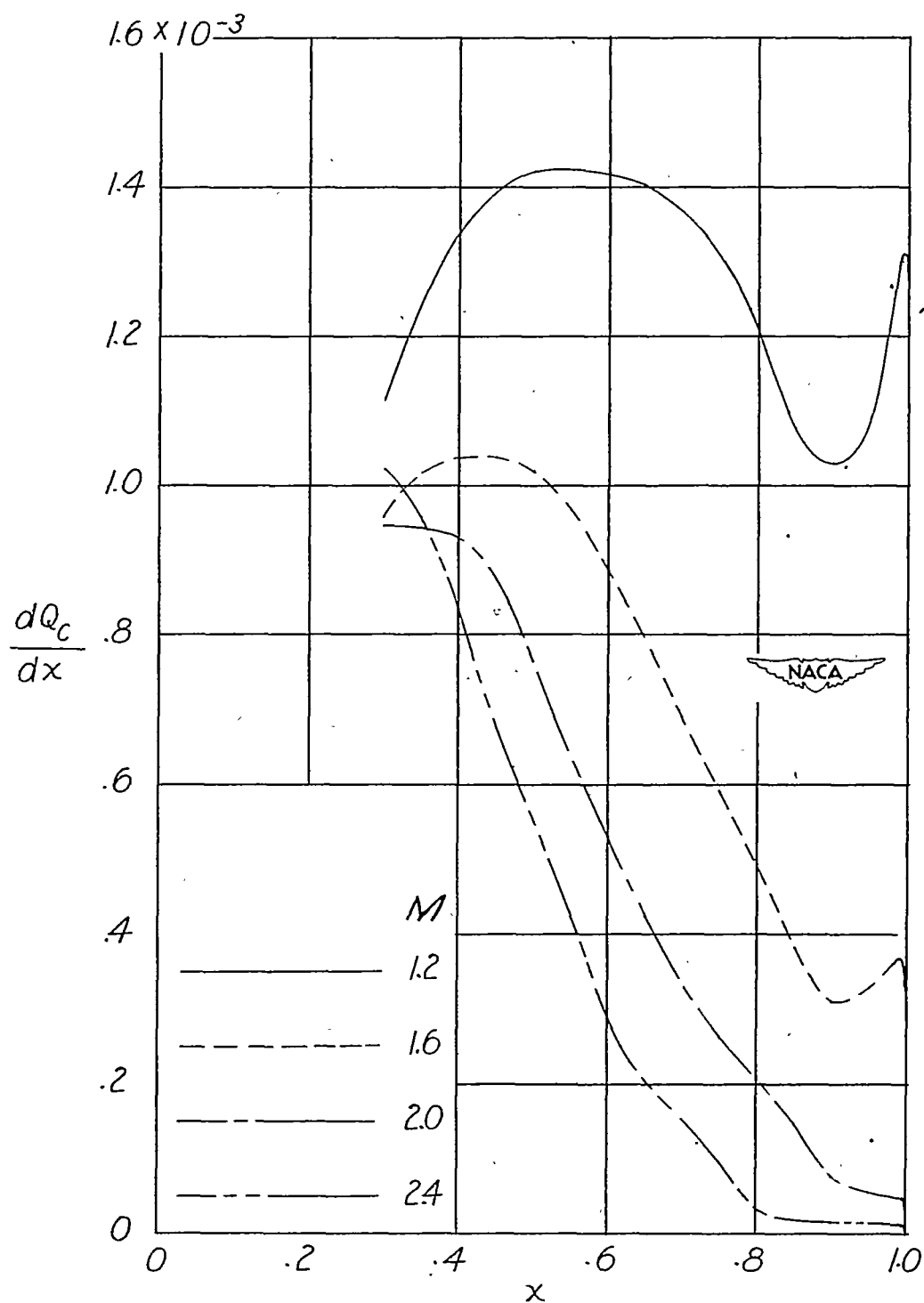
Figure 5.- Torque-gradient curves for propellers of constant centrifugal stress at an advance ratio of 2.0. $\frac{h}{b} = 0.02$; altitude, 40,000 feet; $\sigma = 48,000$ pounds per square inch.
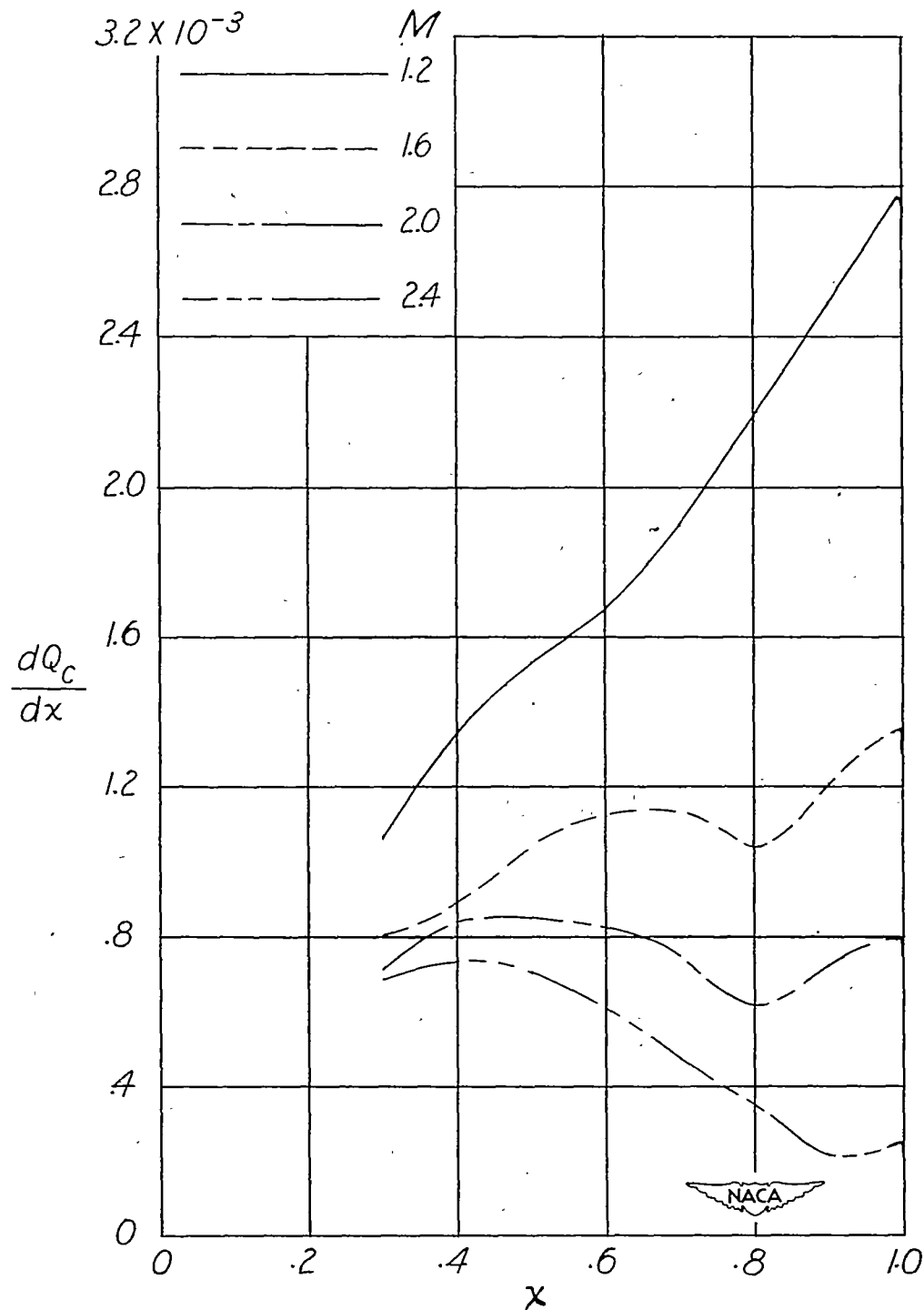
Figure 6.- Effect of Mach number on area distribution for steel
propellers of constant centrifugal stress at an advance ratio
of 3.0. σ = 48,000 pounds per square inch; altitude, 40,000 feet.

Figure 7.- Thrust-gradient curves for propellers of constant centrifugal stress at an advance ratio of 3.0. $\frac{h}{b} = 0.02$; altitude, 40,000 feet; $\sigma = 48,000$ pounds per square inch.
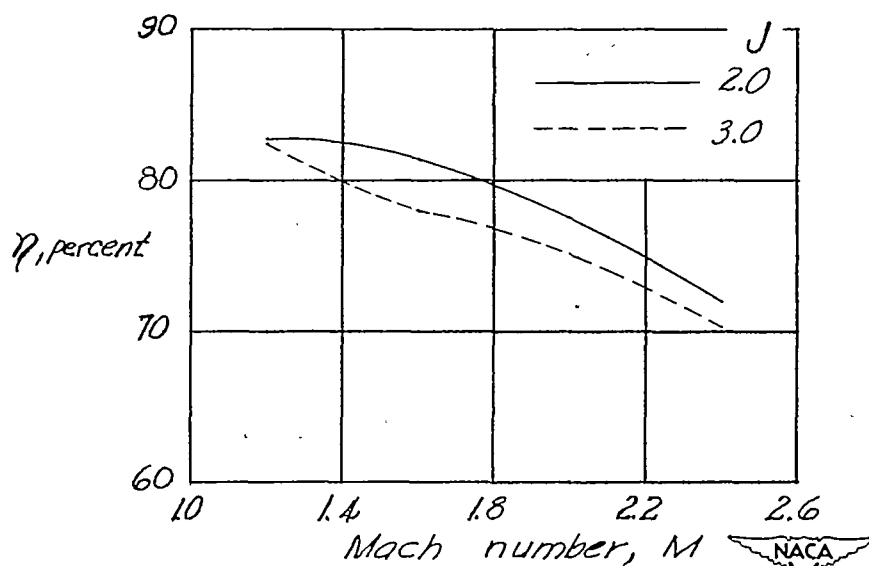
Figure 8.- Torque-gradient curves for propellers of constant centrifugal

stress at an advance ratio of 3.0. $\frac{h}{b}$ = 0.02; altitude, 40,000 feet;

σ = 48,000 pounds per square inch.

Figure 9.- Efficiency for propellers of constant centrifugal stress.
$\frac{h}{b}$ = 0.02; σ = 48,000 pounds per square inch; altitude, 40,000 feet.

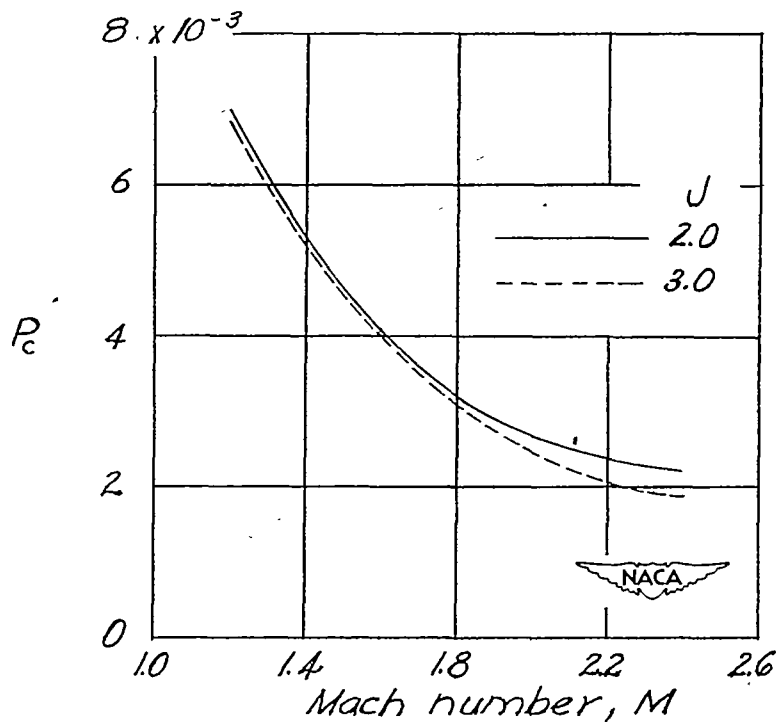Figure 10.- Power absorption for propellers of constant centrifugal stress.
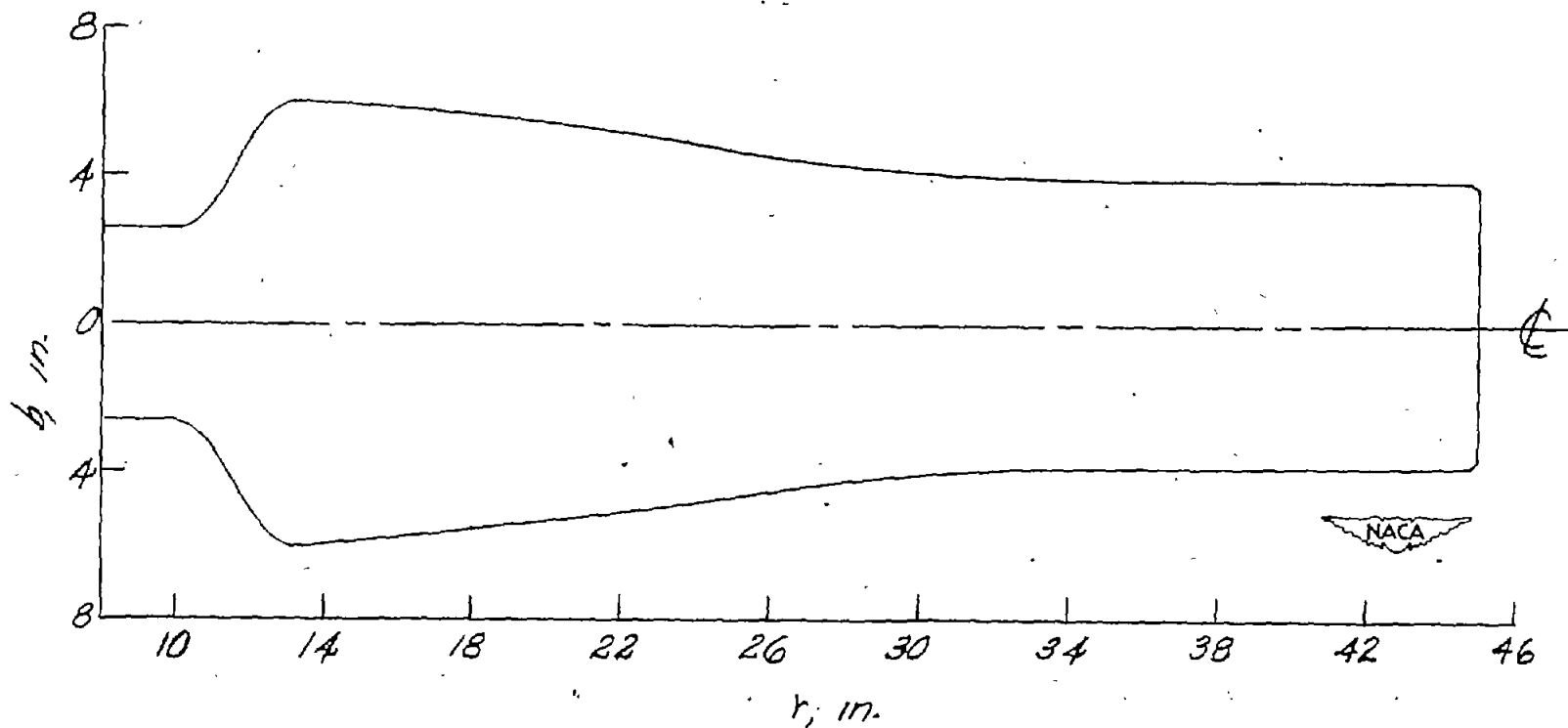$\frac{h}{b}$ = 0.02; σ = 48,000 pounds per square inch; altitude, 40,000 feet.

Figure 11.- Blade plan form for constant-centrifugal-stress steel propeller designed for $M = 0.9$ at 40,000 feet. $\frac{h}{b} \approx 0.02$; $\sigma = 48,000$ pounds per square inch; $J \approx 2.0$.